Chapter 1

The Economy as a Whole in Nineteenth Century Economic Thought

1.1 The Physiocrats

The first systematic attempt to conceptualize the economy as a whole, which had far-reaching influences in all later branches of economic thought was the *Tableau Economique* of Francois Quesnay [Quesnay, 1972 (1758]. The *Tableau* itself is a table with three columns, titled "Productive Expenditure relative to agriculture, etc.", "Expenditure of the Revenue after deduction of taxes, is divided between productive expenditures and sterile expenditure", and "Sterile Expenditure relative to industry, etc.". The table shows how an initial "advance" of 600 livres in productive expenditure (in agriculture, mining, fishing and other sectors now called "primary") and 300 livres in sterile sectors (industry and manufacturing, now called "secondary") will generate a revenue of 600 livres in the productive sectors, of which half is spent on products of the productive sectors and half on the sterile sector. Thiks pattern continues for several rounds of production. The table traces the evolution of the resulting geometric series, resulting in a claimed total "reproduction" of the advances of 900 livres and the revenue of 600 livres.

The *Tableau* is notable from the point of view of the later development of the field economics from several points of view.

First, it is remarkably badly explained and documented and appears to be self-contradictory in some respects. Quesnay also seems to have been prone to simple arithmetic errors (perhaps to show his aristocratic unfamiliarity with an unseemly skill), which make his examples very difficult to follow, and have provoked endless unresolved controversy in the subsequent literature.

Second, we can see the beginnings of the important notion of the sectoriza-

tion of the economy in this schema, a notion which develops its own tradition later, through Marx's schemes of reproduction and Leontief's input output analysis.

Third, the *Tableau* conceptualizes economic process as a sequence of discrete rounds or periods, a conception which turns up later in many places in economics, for example, in the Keynes-Robertson multiplier analysis, but also in input-output analysis.

Fourth, the *Tableau* separates "productive" economic activities from others, and focuses on a particular sub-sector of the economy as "productive", a theme which reappears in much of later economics, and is the foundation of theories of economic growth.

Fifth, the *Tableau* attempts to trace both the production and demand sides of the economy and to account for the sources of demand in the incomes generated from production, which is a fundamental idea in later economic analysis.

Sixth, Quesnay clearly distinguishes between the recovery of the costs of production ("advances") and the generation of a "revenue" in the process. This prefigures the Classical political economists' division of value added among wages, profit, and rent, and Marx's concept of surplus value.

The Classical political economists, however, tended to turn away from a schematic representation of the whole circuit of economic production and sales to an analysis based on a theory of value.

1.2 Adam Smith

Adam Smith's lectures in political economy at Glasgow University [Smith, 1937] synthesized the work of his physiocratic and mercantilist predecessors with his own powerful insights into the dynamics of a just-nascent industrial capitalism. Smith developed a seminal theory of economic growth based on the idea that labor productivity increases with the division of labor, which in turn is limited by the scale of the market, which in turn can be expanded by the accumulation of capital. In Smith Quesnay's distinction between productive and sterile expenditure becomes a distinction between productive and unproductive labor, through which he attempts to analyze the process of capital accumulation. From the point of view of the later development of conceptions of the whole in economics, the more critical and problematic part of Smith's work was his treatment of the theory of value.

1.2.1 The value-added method

The more fundamental and on the whole more successful facet of Smith's theory of value was his resolution of the price of a commodity into wages, profits, and rents through the method of *value added* analysis [Smith, 1937, Book I ch. 6]. Smith argues that the sales price of any commodity in the first instance can be divided into the cost of inputs purchased as commodities from other producers and the "value added" to the commodity by labor, which goes to

pay the wages of the workers engaged in its production, together with rents incurred, the profits appropriated being the residual left over. But these costs in turn generate wages, rents and profits to their producers, together with costs of purchased inputs. Pursuing this logic (note the similarity to the summing of the geometric series in Quesnay) Smith argues that the whole price of a commodity can be resolved into wages, profit, and rent. This logic develops into the modern system of national income accounting based on the value-added principle. If we regard the *net product* of the system as those commodities sold to their final consumers, then the value added in the whole system is equal to the value of the net product. (Note that there is considerable latitude in deciding what categories of sales constitute final sales in this system, but that the value-added principle will be valid whatever conventions are adopted in this regard.

Because we will come across this notion again and again, let us spend a moment here to formalize it. In order to represent an economy with n commodities we will regard bundles (or lists) of the commodities as elements of the mathematical space R^n of *vectors* whose components are n real numbers, $x = (x_1, \ldots, x_n)$. We will always regard vectors as *column* vectors, and write the transpose of a vector x as x^T. We write $x \gg 0$ to mean that *every* component of x is strictly greater than zero, $x \geq 0$ to mean that every component of x is greater than or equal to zero and at least one component is strictly greater than zero (so that $x \neq 0$), and $x \geqq 0$ to mean that every component of x is greater than or equal to zero. The *inverse* of a matrix A, if it exists, is written A^{-1}

A common and fundamental model of production is the *circulating capital* model, in which the inputs to each productive *activity* or *process* are used up in the *productive period*. (We will see later that this model can be interpreted to represent much more general temporal patterns of production.) An activity, or process (words which we will use interchangeably to describe production) consists of a vector $b \in R^n$ of *outputs* of produced commodities which appear at the end of the production period, a vector $a \in R^n$ of input commodities which are required at the beginning of the production period, and a scalar l, representing the amount of labor required to operate the activity. The whole collection of, say, m, processes available to an economy, called its *technology*, can be summarized by matrices $n \times m$ matrices B and A, and a vector $l^T = (l_1, \ldots, l_m) \in R^m$. Each available process is represented by a column of B, the corresponding column of A, and the corresponding component of l^T.

We can represent the *(money) prices* of the commodities by a vector $p = (p_1, \ldots, p_n) \in R^n$ and the (money) wage by a scalar w. If $x \in R^m$ is a bundle of commodities, then according to the rules of matrix multiplication, $p^T x = p_1 x_1 + \ldots + p_n x_n$ is the *value* of the bundle x at the prices p. We can also write $p^T x$ as the *dot product* $p \cdot x$.

In the simplest version of this model we assume that there is just one process that produces each commodity, so that $m = n$, and each process produces just one unit of each commodity, so that $B = I$, the $n \times n$ n-dimensional identity matrix. If we operate the jth process at intensity x_j, the *gross output* of the system will be $x = (x_1, \ldots, x_n)$. To achieve these levels of operation will require

the commodity inputs Ax and a labor input of $l^T x$. The *net product* is then $y = x - Ax = (I - A)x$, since it requires Ax to replace the inputs used up. If prices are p and the wage is w, the value of the gross output is $p^T x$ and the value of the net output is $p^T y = p^T(I - A)x$, the total wages are $wl^T x$, and the total profits $y - wl^T x$.

The direct inputs required to produce the net output y are Ay, and the inputs required to produce those direct inputs are $A^2 y$, so that the total inputs required to produce y are $(A + A^2 + A^3 + \dots)y$. Thus we see that the gross output of a system producing the net output y will be $(I + A + A^2 + \dots)y$. If the matrix A is *viable*, meaning that it is possible to produce a positive net product of every commodity with it, or in mathematical terms, that there exists a vector $x \geq 0$ such that $(I - A)x \gg 0$, then $(I + A + A^2 + A^3 + \dots) = (I - A)^{-1}$. In this way we see Smith's point that $x = (I - A)^{-1}y$, or $y = (I - A)x$, as we saw by considering just one round of production.

1.2.2 Smith's two theories of value

The perhaps less successful aspect of Smith's value theory is his attempt to state a theory of value. In fact he states two different, inconsistent theories of value, thereby generating numerous scholarly works for future generations [Smith, 1937, Book I ch. V, VII–XI].

Smith begins by arguing that the ultimate source of value is human labor expended in production [Smith, 1937, Book I ch. V]. This is Smith's version of the *labor theory of value*. Just what he meant by it has been a subject of controversy every since. One view is that he meant it to be a theory of the *exchange values* of commodities in a "rude and early state of society" before land had been appropriated, and while the means of production were of negligible importance, so that labor was the only input and the only cost of production.

But Smith, after explaining the value-added principle we have discussed, proposes a second theory of value, often called the *adding-up theory of value*. Smith's idea here is to reverse the process of value-added analysis, which shows that the price of a commodity can be resolved ex post into wages, profit, and rent, to generate a theory of the price or value of the commodity by adding up the wages, profit, and rents required to produce it. (In the course of developing this alternative theory, Smith makes the important methodological distinction between *market prices* of commodities, which fluctuate from day to day due to shocks to supply and demand, and *natural prices* of commodities, which are long-term centers of gravity around which the market prices fluctuate.) This project requires Smith to propose theories of the natural wage, natural profit rate, and natural rent. He makes a rather half-hearted stab at each of these problems, but fails to arrive at a rigorous logical theory for any one of them. In discussing the determinants of rents, however, he argues clearly that it is the price of the commodity (say, corn) that determines rent, in the sense that rents of land are high when the prices of commodities produced on the land are high. This line of reasoning (which later economists view as fundamentally correct) unfortunately logically compromises the adding-up theory of value, because the

idea of the adding-up theory was to determine the price of the commodity by adding up the wage, profit, and rent components, but the theory of rent takes the price of the commodity as a determinant of rent, thus rendering the theory circular and logically flawed.

1.2.3 The equalization of the rate of profit and wages

Smith also put forward another seminal idea. He argued that over moderately long time periods both capital and labor would tend to move towards sectors in which their remuneration was highest [Smith, 1937, Book I ch. VIII-X]. Thus labor would tend to move (though not instantaneously) to sectors where the wage was highest, and capital to sectors where the rate of profit was highest. Smith conceptualized this process as a turbulent, gravitational process, which could never succeed in actually equalizing wages and rates of profit across sectors, but would keep them fluctuating around equilibrium levels. In the circulating capital model, the profit rate in the production of commodity, r_j when the wage in the sector is w_j and prices are p, is the ratio of the excess of the purchase price over wage and input costs divided by the capital tied up, which is the input costs together with whatever part of the wages are advanced at the beginning of the period. Thus when the wages are advanced, the profit rate is:

$$r_j = \frac{p_j - p^T a_j - w_j l_j}{p^T a_j + w_j l_j}$$

In order to simplify the algebraic expressions for the profit rate some authors [Sraffa, 1960, for example] assume that the wage is paid at the end of the period, in which case the profit rate is:

$$r_j = \frac{p_j - p^T a_j - w_j l_j}{p^T a_j}$$

If we look at the benchmark equilibrium configuration in which wages are equalized across sectors, so that $w_j = w$ and profit rates are also equalized across sectors, so that $r_j = r$, we see that in the case where wages are advanced the equilibrium prices and profit rate must satisfy the matrix equation:

$$p^T = (1 + r)(p^T A + w l^T) \tag{1.1}$$

In the case where wages are paid at the end of the production period, the prices and profit rate satisfy the equation:

$$p^T = (1 + r)p^T A + w l^T \tag{1.2}$$

The idea of the equalization of the profit rate across sectors is a fundamental element in all notions of economic equilibrium, and a key organizing principle for the Classical political economists.

1.3 Ricardo and the labor theory of value

David Ricardo begins his discussion of the theory of value [Ricardo, 1951, ch. 1] by saying that he agrees with all of Adam Smith's results except for a minor difficulty with Smith's theory of value. Since the theory of value is fundamental to the conception of political economy this is a rather odd way of putting the situation. Ricardo takes exception to Smith's adding-up theory of value on the logical grounds we have just stated, and proposes to return to the labor theory of value as a solid foundation for a theory of distribution and pricing.

I believe that one of Ricardo's motives for making this choice was his desire to use what has since become known as the *corn model* as an analytical tool to understand distribution and accumulation in an industrial capitalist economy. The corn model is crucial to the development of the conception of the whole in political economy. The development of much of general equilibrium theory stemmed from attempts to generalize the corn model.

The corn model is a special case of the circulating-capital model we have just seen in which there is only one produced commodity (corn), so that $n = 1$. The matrix A then becomes the single scalar a, and the condition for productivity is $a < 1$, so that the system can produce a net surplus of corn. Since there is only one commodity, we can also take it as the numéraire of the system, letting $p = 1$, and measure the wage, w, directly as the corn wage. In growth models, the same setup is often used, but with some difference in notation. Discussions of growth models such as Foley and Michl [1999] often use a system of notation where, for example, $x = 1/l$ is the productivity of labor, $\rho = 1/a$ is the productivity of capital, or the output-capital ratio, and $k = x/\rho = a/l$ is the capital intensity of production.

Ricardo embedded this production model in the model of diminishing returns to agriculture and demographic equilibrium put forward by Thomas Malthus [Malthus, 1985]. In this vision, at any point in time the total working population L is (roughly) proportional to the total accumulated capital K (which can be unambiguously measured in terms of corn). The wage w fluctuates around a *subsistence* level, $\bar{w}$, at which the population can just sustain itself, or, if the capital stock and demand for labor is growing, grow rapidly enough to meet the demand for labor. The absolute size of the labor force determines the amount of land cultivated, and the productivity of labor and corn capital on the marginally cultivated land. In the simplest form of this model labor and capital are always combined in the same proportions on land of different quality (a *dose* of labor-capital) so that k remains constant, and $L = K/k$, but labor and capital productivity becomes a function of the total labor force: $x = f(L)$, and $\rho = x/k = f(L)/k$. The *profit rate*, r, for the whole economy is determined by profitability on the agricultural margin:

$$r(K) = \frac{x - w}{k} = \frac{f(K/k) - \bar{w}}{k}$$

The emergence of the appropriate differential rent equalizes the rate of profit on infra-marginal land to the rate of profit on marginal land.

Ricardo makes stunning use of this line of argument to analyze the long-run tendencies of a corn economy in which workers as a class consume their wages, landlords as a class consume their rents, and capitalists as a class accumulate their profits. Under the assumptions, the whole state of the economy at each point in time is summarized by the capital stock, K, which determines the labor force, $L = K/k$, the agricultural margin, productivity of labor and capital, and the profit rate. If capitalists accumulate all of their profits, the law of motion of the accumlated corn capital is:

$$\dot{K} = r(K)K$$

(where $\dot{K} = dK/dt$, the time derivative of the capital stock) which can reach an equilibrium only at the capital stock K^* where:

$$r(K^*) = 0$$

This *stationary state* occurs when the population has expanded to the point where the marginal land yields only just enough corn to pay the subsistence wage. The profit rate in the stationary state is zero, and accumulation stops due, ultimately, to diminishing returns imposed by the finiteness of land resources. Ricardo argued for the advantages of staving off the steady state as long as possible by adopting land-saving technical improvements, and opening up free trade in corn, which has the effect of moving the agricultural margin to the whole world, where productivity of labor is likely to diminish much less rapidly than in any one country.

Ricardo wanted to argue that the conclusions of the corn model were also applicable to an industrial economy with many produced commodities. The seeds of future investigations of *general equilibrium* theories lie in the logical problems Ricardo confronted in defending this generalization against the criticisms of others, particularly Malthus. Ricardo begins his defense by adopting a rigorous version of the labor theory of value. If the prices of all commodities were proportional to the quantities of labor embodied in them, then it would be possible to reduce all commodities to quantities of a single uniform labor. Clearly in this case labor can take the place of corn in the corn model and the logic of the corn model will go through.

In particular the labor theory of value would underpin the important analytical insight from the corn model that distributional shares were, at least with given technology, *antagonistic*, in the sense that wages, profit, and rent have to come out of a given magnitude, the value of the commodity in terms of embodied labor. Thus one of the distributional shares can increase only at the expense of a decline in the others. For example, in an economy without land, there is a *wage-profit rate* tradeoff, a schedule of possible combinations of wages and profit rates which cannot be circumvented. One can see this transparently in the corn model, where the profit rate on the marginal land clearly varies inversely with the corn wage.

Unfortunately, as Ricardo recognized, prices proportional to embodied labor (even if one can surmount the problem of finding a common denominator

through which to aggregate different types of labor) will not equalize the profit rate in different sectors of production unless by accident (or, as it is often expressed in the literate, a *fluke*) the proportions in which all the different commodities enter as inputs to the production of each particular commodity are the same, and the periods of production for which the input commodities are required are also the same. (In the circulating capital model the periods of production in all sectors are assumed to be the same.) Malthus pointed out that this was very unlikely. Another possible way to generalize the corn model argument would be to assume that only corn and labor are required as an input to corn production, and that the subsistence wage bundle also consists only of corn. While there was some plausibility to this assumption in the conditions of early nineteenth-century agriculture, in which the bulk of the subsistence of workers consisted of bread and beer, it was clear that the assumption was not precisely correct (workers need manufactured commodities both for consumption and as means of production).

Ricardo grappled with this difficulty to the end of his life. He seems to have had two main strategies for answering these criticisms. The first was to argue that while the labor theory of value in the sense that prices were proportional to embodied labor was not strictly true, it still might be approximately true. (This position was satirized by George Stigler as the "93% labor theory of value".) The second was to seek a single *invariable standard of value*, a commodity or bundle of commodities that would have the property that its price would not vary with a change in distribution. If such a commodity existed, Ricardo could use it as the benchmark through which to analyze the determination of the profit rate with accumulation of capital, along the lines of the corn model, and then deal with the problem of the pricing of particular commodities in terms of their deviations from the invariable standard of value. Unfortunately (or inevitably) Ricardo was unable to find a satisfactory invariable standard of value, leaving the generalizability of the corn model undecided.

1.4 Marx

The political economic work of Karl Marx took the form of a critique of the Classical political economists. Two of Marx's numerous fertile discussions of the theories of the Classical political economists have had particular influence on twentieth-century developments of conceptions of the economy as a whole, his treatment of the problem of equalization of profit rates (often called the *transformation problem*) and his late work on schemas of reproduction in capitalist economies. Much of the rest of Marx's work casts a long shadow over twentieth-century economics, from his theory of surplus value as arising in the exploitation of labor, to his theories of induced technical change, the tendency for the rate of profit to fall, and the long-run fate of capitalism as an economic system, but these issues are somewhat tangential to our present concerns.

1.4.1 Equalization of the rate of profit in Marx

Marx saw the labor theory of value as enunciated by Ricardo not so much as a correction of Smith's logical slips, but more as a reflection of the more general historical-philosophical point of view of historical materialism. This powerful insight, which Marx shared with his collaborator Friedrich Engels, argued that the key to understanding human history and social change is to understand the mechanisms in any society through which the social surplus product is appropriated and controlled. Since the ultimate source of surplus product is human labor, organized in some form or other, the question of control over the social surplus product always boils down to the question of the organization and control over human labor time. In this perspective Ricardo's theory of value and distribution fits as a particular, extraordinarily well-developed analysis of the appropriation of human labor time in capitalist societies. The labor theory of value for Marx, in this way of looking at things, was not so much a technical device for solving problems of value and distribution as a way of connecting the operation of capitalist society with the general sweep of human history.

But Marx also accepted the "long-period" method of the Classical political economists, and saw the power of Smith's theory of the tendency for profit rates to equalize. He characteristically sought to reformulate the Classical analysis critically to be consistent with his own analytical framework, but tells the same theoretical story. Thus Marx recognized and tried to grapple with the inconsistency between the labor theory of value, which sees the source and substance of the value of commodities in the labor expended in their production, and the principle of profit rate equalization, which means that surplus value tends to be proportional to the value of capital invested in a line of production, not to the quantity of labor employed in it. This is the the main subject of Marx [1981, Parts I and II]. These notes are preliminary and were never fully revised for publication by Marx, though they were written before the publication of Marx [1976]. As a result there are many gaps and leaps in his argument which have provided rich material for later scholarly and political dispute.

Marx seems not to have been attracted by Ricardo's somewhat evasive argument that ratios of embodied labor coefficients might be a pretty good approximation to relative natural prices. Instead, he argues that competition among capitalists can only have the effect of redistributing surplus value, not of changing its magnitude, nor the magnitude of social labor time.

These ideas can be conveniently expressed in terms of the circulating capital model of production. If $\lambda = (\lambda_1, \ldots, \lambda_n)$ is the vector expressing the labor embodied in the n commodities, it must satisfy the equation:

$$\lambda^T = \lambda^T A + l^T$$

It is worth noting that this equation is the same as either equation (1.1) or (1.2) (substituting λ for p) when $r = 0$ and we take $w = 1$, so that labor itself becomes the numéraire. In this simple case of no joint production and purely

circulating capital, the embodied labor coefficients can be solved for as:

$$\lambda^T = l^T(I - A)^{-1} \tag{1.3}$$

When $r > 0$, however, $\lambda \neq p$ in general (unless the structure of productive inputs in all the sectors is identical, in which case all the commodities are the same from a production point of view and we are effectively back in the case of the corn model). Or, to put it another way, if prices remain proportional to the embodied labor coefficients, the profit rates in different sectors will not be equalized. Marx argues that prices will adjust to redistribute surplus value so as to equalize profit rates. His tabular schemas can be represented algebraically as determining prices $\hat{p}$ that satisfy the equations:

$$\hat{p} = (1 + r)(\mu\lambda^T A + wl^T)$$

Here μ is a scalar that translates social labor time into money, in order to make the units of the right- and left-hand sides of this equation consistent. The appearance of the embodied labor coefficients λ in the expression for the costs of inputs to production makes it difficult to interpret this equation as a complete representation of long-period equilibrium in the Classical political economic sense, as Marx may or may not acknowledge in his rather cryptic remarks on this problem, but as later commentators, mostly rather hostile to the labor theory of value and Marx's theory of exploitation, did not fail to emphasize.

Marx goes on to argue that the important thing about the "transformation" of embodied labor coefficients into profit rate-equalizing prices of production is the conservation of various aggregate expressions of labor time, such as the "total value", the value-added, and the aggregate surplus value. In this sense Marx appears to want to use the "total value", $\lambda^T x$, or the value of the net product $\lambda^T y = l^T x$, to play the analytical role of Ricardo's invariable standard of value.

The later literature on this problem is immense, and takes us somewhat far afield from our main themes. But Marx's statement and analysis of this problem did have fundamental influence on the twentieth-century conceptions of the economy as a whole developed by those scholars who studied and knew Marx, expecially Piero Sraffa [Sraffa, 1960].

1.4.2 Marx's schemas of reproduction

In Marx's last major analytical effort, undertaken despite his ill-health and possibly depression at Engels' urging, he tackled the problem of understanding how a capitalist economy could realize, or sell, the commodities it produced [Marx, ????, ch. XXI and XXII]. (A more complete discussion of Marx's schemas of reproduction can be found in Foley [1986, ch. 5].)

For the purposes of understanding the flows of value in a capitalist system, Marx divided productive activities into two *Departments*, Department I which produced the means of production and Department II which produced means

of subsistence. These Departments are not quite the same conceptually as the *sectors* we introduced in the circulating capital model. The sectors correspond to commodities which are distinguished (basically) by the structure of their productive technology, while Marx's Departments correspond to the function which commodities play as inputs to production. Thus the same commodity which would appears in one sector in the circulating capital model, might appear in both of Marx's Departments if it is used both as a means of production and as a means of subsistence.

Despite this conceptual difference, it is possible to express the kernel of Marx's analysis of reproduction in the circulating capital framework. An economy undergoing "simple reproduction" just reproduces its inputs, including workers' consumption, with the entire surplus being consumed by capitalists. If we denote by b the vector representing workers' subsistence consumption, and by d the vector representing capitalist consumption, an economy in simple reproduction must satisfy the equation:

$$x = Ax + bl^T x + d$$

The vector product bl^T is a $n \times 1 \times 1 \times n = n \times n$ matrix representing the commodities required to meet workers' subsistence needs. The columns of the *augmented input matrix* $\hat{A} = A + bl^T$ represent the total inputs required for production of one unit of each commodity, including the induced demand for workers subsistence. An economy in simple reproduction, then must operate in definite input proportions:

$$x = (I - \hat{A})^{-1} d$$

If the economy is undergoing *expanded reproduction*, some part of the surplus value is being re-invested by capitalists, but in such a way that both Departments (or all sectors in the circulating capital model) are growing uniformly at a rate g. In expanded reproduction the economy must reproduce not just the inputs (including workers' subsistence) but enough more to allow production at an expanded scale in the next period, so that:

$$x = (1 + g)\hat{A}x + d$$

Marx took considerable trouble to work out the proportions required for simple and expanded reproduction in particular examples through a process of arithmetical trial-and-error. This work had a powerful influence on twentieth-century conceptions of the economy as a whole through the work of Wassily Leontief and John von Neumann.

1.5 Walras, Jevons, Fisher, and the marginalist revolution

The last important strain of nineteenth-century economic thinking that is central to our story is the *marginalist revolution*. There had always been discomfort with the labor theory of value and its cost-based theories of natural

price, because the actual experience of market exchange conveys the lesson that commodities sell at whatever someone is willing to pay for them. (The Classical political economists understood this, of course, but argued that there were deeper forces tending to organize the prices at which commodities, at least easily reproduced commodities, would exchange.)

The marginalist revolution, which emerged more or less simultaneously in Britain, France, and Austria in the work of Stanley Jevons, Carl Menger, and Léon Walras, and was rapidly developed in the United States and Italy by Irving Fisher and Wilfredo Pareto, was based on the idea that subjective valuation could serve as a foundation for price theory if it was recognized that *marginal* rather than *total* utility corresponded to market price.

The canonical set-up for the marginalists is the maximization of some *utility*, or, in less loaded language, *objective function*, $f(x)$, where $x \in R^n$ is a vector of final consumption, constrained by the availability of resources and technology, represented as a constraint $g(x) \leqq b$, where $g : R^n \to R^m = (g_1(x), \ldots, g_m(x))$ is a vector-valued function mapping points in consumption space, x to points in resource space $g(x) \in R^m$. This constrained maximization problem, which will be central to our later discussions in various contexts, is:

$$\begin{aligned} &\max_{x \geqq 0} f(x) \\ &\text{subject to} \\ &\quad g(x) \leqq b \end{aligned} \tag{1.4}$$

A constrained maximization problem of this type has a related *Lagrangian*, which is a function of the choice variables $x \in R^n$ and a vector of *Lagrange multipliers* or, more appropriately in an economic context *shadow prices*, $\mu = (\mu_1, \ldots, \mu_m) \in R^m$:

$$\mathcal{L}(x, \mu) = f(x) - \mu^T (g(x) - b)$$

A *saddle-point* of the Lagrangian function is a pair of values (x^*, μ^*) with the property that x^* *maximizes* $\mathcal{L}(x, \mu^*)$ for $x \geq 0$ and μ^* *minimizes* $\mathcal{L}(x^*, \mu)$ for $\mu \geq 0$, or:

$$\mathcal{L}(x, \mu^*) \leq \mathcal{L}(x^*, \mu*) \leq \mathcal{L}(x^*, \mu) \text{ for all } x \geqq 0, \mu \geqq 0 \tag{1.5}$$

It is not hard to prove that a saddle-point of the Lagrangian of such a system actually solves the constrained maximization problem, though the converse is not always true, as we shall see. It is also not hard to prove that a saddle-point satisfies the *first-order conditions* (though the converse is not always true,

either):

$$\begin{aligned} \frac{\partial \mathcal{L}(x^*, \mu^*)}{\partial x} &= \frac{\partial f}{\partial x} - \mu^T \frac{\partial g}{\partial x} \leqq 0 \\ \text{and } (\frac{\partial f}{\partial x} - \mu^T \frac{\partial g}{\partial x}) x^* &= 0 \\ \frac{\partial \mathcal{L}(x^*, \mu^*)}{\partial \mu} &= -(g(x) - b) \geqq 0 \\ \text{and } {\mu^*}^T (g(x) - b) &= 0 \end{aligned} \tag{1.6}$$

The vision of the marginalists was to identify the shadow-prices in some maximization problem of this kind with market prices, thereby interpreting market prices as a measure of the *social scarcity* of the various resources.

The central problem in this project is that in an economy there are *many* different subjective valuations, corresponding to the different consuming units, not just one. This difficulty lies at the heart of many of the formal difficulties with the marginalist approach to conceiving of the economy as a whole.

Léon Walras, whose work had almost no impact on the development of political economy and economics in its own time, attempted to solve these problems by conceptualizing the economy as a collection of consumers each maximizing their individual objective functions subject to a budget constraint defined by parametrically given prices, and a collection of firms, each maximizing profit at the same parametrically given prices, and seeking a system of non-negative prices at which these separate maximization problems might have a mutually consistent set of solutions. Walras ran into several important problems in pursuing this project. First, he saw that some prices might have to be zero at competitive equilibrium, raising the problem of which commodities will be free and which priced. Second, he found that it was impossible to satisfy the condition of equalization of the rate of profit on all capital goods for an arbitrary initial endowment of capital goods, an issue we will discuss further. Third, he was unable to give a satisfactory mathematical argument for the existence of a non-negative price system that would make the plans of the households and firms consistent, and had to confine himself to the weaker observation that there were the same number of determining conditions to be met as prices to be determined.

The marginalist concept has demonstrated an almost mesmerizing power over the minds and imagination of a certain group of analytically-oriented economists. It appears to offer the hope of a completely general and consistent methodological approach to any economic, and, indeed, in some people's minds, any social, problem. A tremendous explosion of analytical work followed from this basic insight, mostly devoted to generalizing the commodity space in which the maximization problem is situated (so that the commodity vectors might be infinite sequences of time-dated commodities, or functions, or random variables). Unfortunately, this concept also seems to engender a hostility and intolerance for other theoretical approaches to the conceptualization of economic relations as a whole, as well, an attitude which sits uncomfortably with

the general ideals of skeptical open-mindedness in science.

A good part of the story of twentieth-century economic theory consists of attacks on the problem of how to reconcile Classical notions of long-period equilibrium with the marginalist vision, a problem that has not really been satisfactorily resolved.

1.6 Thermodynamics and economics

There are striking parallels between the formalism of marginalist economics and those of physical thermodynamics [see Mirowski, 1989]. Thermodynamics studies systems, for example, gases, confined to a finite volume, in terms of paired *intensive* and *extensive* variables. Extensive variables scale with the size of the system, while intensive variables do not. For example, the pressure of a gas is an intensive variable, while the volume or the number of molecules is an extensive variable. Bringing two systems with the same intensive variables into interaction leads to a combined system with the same intensive variables, but with extensive variables equal to the sum of the extensive variables of the two systems. Other examples of intensive/extensive variable pairs in physical thermodynamics are temperature/entropy and chemical potential/number of molecules. (The chemical potential is a measure of the tendency for molecules to enter or leave the system.) Furthermore, it is possible to demonstrate formally that intensive/extensive variable pairs correspond to shadow price/resource constraint pairs in a formulation of the thermodynamic problem as a constrained maximization problem.

The parallel with the marginalist vision of the explanation of price in economic systems is striking. The intensive variables for the economic system are prices, the extensive variables the total endowment of the various commodities. If one brings two economies with the *same* prices into interaction, say, through international trade, there will be no trade and no change of prices, but extensive variables such as the total endowment, or GDP, will increase.

One striking difference between the Classical political economists and the marginalists (and their neoclassical and general equilibrium theory descendants) is that Classical political economy argues much more frequently in terms of intensive variables like the profit rate, or the ratio of surplus value to wages, which do not depend on the absolute scale of the economic system, than in terms of extensive variables. The one great exception to this observation is the connection Ricardo and Malthus saw between the absolute size of the economy as measured by its capital stock or population and the rent on land due to diminishing returns to land. Rent/land is an intensive/extensive variable pair similar to pressure/volume in a thermodynamic system.

Thus one of the methodological bifurcations between the Classical and neoclassical schools is the tendency for Classica/Marxian analysis to be carried on in terms of an analysis of intensive variables that are independent of the absolute scale of the economy, and to seek explanations for intensive variables like prices without reference to the total endowments of resources. The neoclassical

school, on the contrary, instinctively sees the economic problem as involving a full set of dual intensive/extensive variables and distrusts analyses that neglect any subset, such as the extensive variables. For the neoclassicals there is no hope of explaining prices without reference to absolute endowments of scarce resources, while for the Classically-oriented economist explanations of price based on endowments seem thin and unconvincing.

This, too, will be a major theme in the unfolding story of twentieth-century economic theory.